

	Team Control Number	
For office use only	33534	For office use only
T1 _____		F1 _____
T2 _____		F2 _____
T3 _____	Problem Chosen	F3 _____
T4 _____	D	F4 _____

---

**2015 Mathematical Contest in Modeling (MCM) Summary Sheet**

(Attach a copy of this page to each copy of your solution paper.)

**Summary**

Facing the fact that the population is increasing but the resources on earth is limited, we have to find a sustainable way to manage our development. Before planning for sustainable development, we have to find a standard to judge whether a country is sustainable or not. To address this problem, we select seven indicators to show the level of sustainable development of a country. Combining with the Analytic Hierarchy Process (AHP), we calculate the weight of different indicators in showing the level. Based on the weigh vector of the seven indicators, we put forward an algorithm to obtain scores of some countries to estimate the level of sustainability. We use those scores to set up a Fuzzy mathematical model in order to find a boundary to differentiate whether a country is sustainable or not, meanwhile we checked our model with several countries' data and the result shows a good consistency to reality.

Once we can judge whether a country is sustainable or not, we select Nepal from the 48 least developed countries and give a score by adopting our sustainable development model. We use data of seven indicators to analysis and provide some specific programs and policies. By applying our plan and asking help from ICM, we hope it will be able to help Nepal to increase its level of sustainability. But here comes another difficulty: How can we predict the change by implementing our plan in Nepal over several years? We establish a regression model to answer this question. Meanwhile we use a random variable to describe the environmental factors. Then We optimize the strategies for ICM by giving an algorithm based on the assuming that ICM only aid to ameliorate one indicators each year. In this algorithm, we give the concept of 'the most valuable' and combining our predicting model, we can easily help ICM to select a most effective way to aid Nepal.

Is it sustainable?

# Contents

<b>1</b>	<b>Introduction</b>	<b>4</b>
<b>2</b>	<b>How to evaluate the sustainability of a country?</b>	<b>5</b>
2.1	Assumptions . . . . .	5
2.2	Indicators of sustainable development . . . . .	5
2.3	Model and algorithm . . . . .	7
2.3.1	Sustainable development model . . . . .	7
2.3.2	Algorithm . . . . .	8
2.3.3	How to identify whether a country is sustainable or not . . . . .	9
2.4	Model simulation . . . . .	9
<b>3</b>	<b>How can Nepal become more sustainable?</b>	<b>10</b>
<b>4</b>	<b>Prediction and optimization</b>	<b>12</b>
4.1	Prediction of our plan for Nepal . . . . .	12
4.2	How to select a effective plan for ICM . . . . .	16
4.2.1	Assumption . . . . .	16
4.2.2	Select a highly effective plan for ICM . . . . .	16
<b>5</b>	<b>Strength and weakness</b>	<b>17</b>
5.1	Sustainable development model . . . . .	18
5.2	Prediction and Optimization model . . . . .	18
<b>6</b>	<b>Conclusions</b>	<b>19</b>

# 1 Introduction

Facing the problem that human need strike a balance between development and the health of earth, people have paid attention to sustainable development which is defined by the 1987 Brundtland Report as "development that meets the needs of the present without compromising the ability of future generations to meet their own needs. In this paper, we develop three models to evaluate sustainability of a country and predict the sustainability of a country in the future. Our problems can be divided into these three sub-problems:

- **How to evaluate the sustainability of a country?**

To estimate the sustainability of a country, we need to use several indicators. Besides, the Analytic Hierarchy Process (AHP) has the special advantage in this problem. So we think developing our model based on AHP is appropriate. Then we figure out a set of scoring criteria with the weight vector to evaluate the sustainability of a country. Then we use fuzzy mathematical model to find a boundary to distinguish whether a country is sustainable or not.

- **The plan for Nepal:** Based on the sustainable development model we give, we can calculate the score of Nepal which is selected from the 48 least developed countries. Analysing the 7 different indicator's data and the situation in Nepal, we give a plan for Nepal including some policies and programs to help them enhance sustainability. Besides, based on the economics of Nepal, we also give some advice to find help from ICM in Nepal such as finance and technical aid.

- **Prediction and optimization:** In order to predict the change in Nepal over 20 years with our plan assisted by ICM, we establish a regression model to fit the data in the past 15 years. By changing the regression coefficient in the regression model based on the plan we put forward, we make a prediction of the change in Nepal over 20 years. Meanwhile we use  $\varepsilon$  to represent the random deviation caused by environmental factors such as natural disaster and war etc. Apart from prediction, we also give an algorithm to calculate the most effective plan for ICM.

## 2 How to evaluate the sustainability of a country?

Sustainability of a country can be used to indicate social economic, environment development, and the ecosystem's health. But the problem needed to be solved is that how can we evaluate the sustainability of a country since the concept of sustainable development is too abstract to identify. To solve this problem, we select several indicators and use Analytic Hierarchy Process (AHP) which is created by Satty in 1982 to get a score based on those indicators to quantize the sustainability of a country. In this process, we use US as standard country to get other country's score. We may distinguish the sustainability between different countries by contrast their scores, and ICM will be informed to aid those countries with low score.

### 2.1 Assumptions

We may need following assumptions:

- We assume that the score and the value of indicators are linear relationship. As a result, we could use the condition in U.S. as a reference to assess score in other countries, specially, high score means the country have a good level of sustainable development.
- We assume that every country develops in a stable way. In the process of scoring, we ignore the fact that there will be some natural disasters such as earthquake and tsunami in developing process. We have to acknowledge that the natural disasters are event of small probability, consequently, it is unlikely to happen in the developing process.
- For simplicity, we only consider seven major indicators as the main standards to reflect the level of sustainable development. Although other less persuasive factors can be used to show the sustainability, they are ignored in order to simplify the calculating.

### 2.2 Indicators of sustainable development

- **GNI per capita (G):** The Gross National Income (GNI) is the total domestic and foreign output claimed by residents of a country, consisting of Gross Domestic Product (GDP) plus factor incomes earned by foreign residents, minus income earned in the domestic economy by nonresidents.[2] We consider GNI per capita as one of the indicators because the GNI per capita index can show the economics of a country, while the economic status can show the level of the sustainable development.
- **Access to an improved water source(% of rural population with access) (A):** It refers to the percentage of rural population using an improved drinking water source. The improved drinking water means water protected from contaminate. The percentage of rural population to get the clean water can be a good indicator to show sustainability for the reason that accessing to the clean water is premise

of good health, which can reflect the social health condition as an aspects of sustainable development.

- **Ecological Footprint (EF):** Ecological Footprint Accounting addresses whether the planet is large enough to keep up the demands of humanity.[3] Ecological Footprint will be a feasible factor to judge whether a nation has high sustainability. If a nation has a low EF value, it means the country tend to have good environment protection and have varieties method to access to energy, which could use to show whether a country is sustainable or not.
- **$CO_2$  emissions index (C):**  $CO_2$  emissions index data is the total  $CO_2$  emissions in the nation divide by GDP, this can be a persuasive evidence to indicate condition of sustainability of a nation. If a nation have low  $CO_2$  emissions index, we may deduce that this country will have a good policy on limit the usage of greenhouse gases. Since if a country has a low level of  $CO_2$  emissions index, the amount of  $CO_2$  emissions released when producing unit value is in a low level, therefore, we can infer that this country has a good sustainability.
- **Life expectancy at birth (L):** Life expectancy at birth can measure and predict a person's longevity. We can give a speculation on the level of sustainable development depending on life expectancy. It can reflect the condition of public health and the medical system, and a good medical system shows a high sustainability of a country.
- **Forest area (% of land area) (F):** Many forests are destroyed for economic purpose now. People in many country cut down numerous valuable trees without taking the sustainable development into consideration. In fact, the percentage of forest area of land area can reflect whether country adopt sustainable policies, for the reason that sustainable development would not develop economics at the cost of environment, consequently, we use the forest area (% of land area) as an indicator of evaluating the sustainability of a country.
- **Alternative and nuclear energy (% of total energy use) (AE):** The alternative energy include wind, solar, hydro and other forms of alt energy. Alternative and nuclear energy are both clean energy which means they will not produce carbon dioxide when used. Hence, the percentage of alternative and nuclear energy in total energy use can be a good index to show the sustainability of a country which has been defined as developing without compromising the ability of future generation to meet their own need.

These indicators above are shown in Figure 1

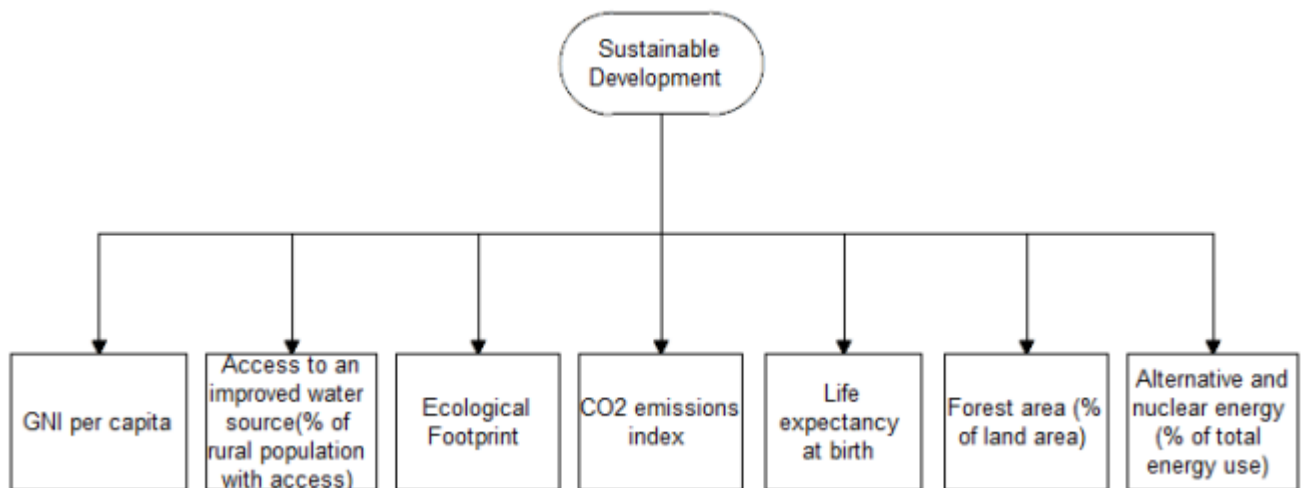


Figure 1: Indicators of sustainable development

## 2.3 Model and algorithm

### 2.3.1 Sustainable development model

By building our model, we want to identify whether a country is sustainable or not. So we can call our model—**sustainable development model**.

We choose Analytic Hierarchy Process (AHP) to help us to distinguish the sustainability between different countries, to utilize the AHP algorithm, we need to set up a  $7 \times 7$  reciprocal matrix (table 2) with comparing different indicators, and the meaning of the number has shown in table 1.

Table 1:the meaning of the scales

Intensity of Value	Interpretation
1	Requirements i and j have equal value.
3	Requirement i has a slightly higher value than j.
5	Requirement i has a strongly higher value than j.
7	Requirement i has a very strongly higher value than j.
9	Requirement i has an absolutely higher value than j.
2,4,6,8	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement i has a lower value than j.

Subjectively, the number in the Table 2 is determined by our experience.

Table 2: the reciprocal matrix

	G	A	EF	C	L	F	AE
G	1	1	2	1	1	1	1/2
A	1	1	2	1	1	1	2
EF	1/2	1/2	1	1/2	1	1/2	1/3
C	1	1	2	1	1	1	1/2
L	1	1	1	1	1	1	1/2
F	1	1	1/2	1	1	1	1/2
AE	2	2	3	2	2	2	1

**Note:** G, A, EF, C, L, F, AW refer to the seven indicators in §2.2.

We can get the weigh  $\omega_i$  by using matlab, and the specific data are shown in table 3.

Table 3: The weigh of different indicators

Indicators	G	A	EF	C	L	F	AE
weight	0.1299	0.1810	0.0759	0.1299	0.1195	0.1143	0.2494

The analysis of consistency: to use AHP, we need to assess the consistency of the date.

- We can calculate the maximum eigenvalue of the matrix by matlab, the result  $\lambda_{max}$  is 7.32.
- The consistency index can be calculated by the formula :

$$CI = \frac{\lambda_{max} - n}{n - 1} \quad (1)$$

$n$  is the order of the reciprocal matrix, in this case,  $n$  is 7, Then we get  $CI = 0.053$ .

- To ensure data have a good consistency we need Consistency Ratio(CR) under 0.1, and CR can be calculate by the formula:

$$CR = \frac{CI}{RI} \quad (2)$$

**Note :**  $RI$  is 1.32 when  $n = 7$  [5].In this case, we get  $CR = 0.0408$ .

Above all, we can see our data have a good consistency.

### 2.3.2 Algorithm

After we get the weigh of different indicators, we can get a score by using formula shown below:



$$score = \frac{G}{G_1} \times \omega_1 + \frac{A}{A_1} \times \omega_2 - \frac{EF}{EF_1} \times \omega_3 - \frac{C}{C_1} \times \omega_4 + \frac{L}{L_1} \times \omega_5 + \frac{F}{F_1} \times \omega_6 + \frac{AE}{AE_1} \times \omega_7 \quad (3)$$

**Note:**

- In the formula above,  $G \dots AE$  is the indicator of country being assessed,  $G_1 \dots AE_1$  is the data of U.S. in 2007 which we used as critical standard, the specific data will be shown in table 4 later.
- $\omega_i$  is the weight of the  $i$  th indicator.
- Considering the fact that the large value of EF and  $CO_2$  emissions index indicate the country have a bad performance in sustainable development. As a consequence, higher the EF or  $CO_2$  emissions data of a country, lower it gains in the score.
- We assume that score and the value of indicators are linear relationship, this can be support by the statics given by the world bank[6].

**2.3.3 How to identify whether a country is sustainable or not**

Using the algorithm above, we can easily get a score which can show the level of sustainable development for each country, and by comparing the score, we can clearly know whether a country have a good sustainability or not.

We use fuzzy mathematics model to determine a boundary to distinguish whether a score is sustainable enough. We let Fuzzy set A represent 'sustainable', and we give the subordinative function of A as

$$f(x) = \frac{1}{1 + \left(\frac{0.5}{x+0.05}\right)^2} \quad (4)$$

We can use  $f(x)$  to calculate the degree of sustainability of a specific score. For example, the score of Nepal is 0.29,  $f(0.29) = 0.316$  which means the degree of sustainability of Nepal is 0.316, and the score of US is 0.59,  $f(0.59) = 0.621$  which means the degree of sustainability of U.S. is 0.621. By solving the equation:

$$\frac{1}{1 + \left(\frac{0.5}{x+0.05}\right)^2} = 0.5 \quad (5)$$

we get  $x = 0.45$ . Here  $x$  is the cut-off point in the Fuzzy set A satisfy  $f(x) = 0.5$ . As a result, we use 0.45 to distinguish whether a score is 'sustainable' or not.

**2.4 Model simulation**

To test the validness of our model, we use the data of several countries. We use the original data from World Bank to calculate the seven indicators in §2.2, and we get a good result. The value of seven indicators of these countries are shown in Table 4.

Table 4: The data of seven indicators and the score

	G	A	EF	C	L	F	AE	score
United States	48690	91.7	8.0	4.0229	77.8390	33.0578	10.8180	0.5883
United Kindom	46110	100	4.9	1.7493	79.4487	11.8191	8.2289	0.5700
Nepal	380	82.1	3.6	2.6188	65.7200	25.3644	2.5711	0.2922
Angola	2550	36.4	1.0	4.1579	49.4357	47.2081	2.0074	0.2204
Australia	37330	100	5.4	4.4201	81.2926	19.7951	1.3577	0.3275
Bangladesh	560	80	0.65	6.0878	68.2831	11.1377	0.2419	0.1055
Brazil	6100	81.3	2.5	2.6573	72.2629	62.1916	15.1084	0.7415
Canada	41230	99	6.8	3.8467	80.3698	34.1049	20.6978	0.8351
Congo, Dem. Rep	260	28.2	0.9	1.5910	48.2135	68.4013	3.1356	0.3793
Japan	37660	100	4.2	2.8719	82.5070	68.4570	15.3047	0.8815
Sudan	890	51.9	1.7	2.1633	60.6625	29.5082	0.8159	0.2328
Tanzania	410	44.5	1.3	3.6548	55.9876	39.1038	1.1796	0.2069

**Note:**

The physical dimension of the each data above has been ignored for the reason that we only concern about the ratio between its original data and the data of U.S..

We can get the score of those countries mentioned above by using the formula (3). From the result shown above, we can learn that our model are basically accord with the fact.

### 3 How can Nepal become more sustainable?

We can know the score of Nepal in 2007 is 0.2922(in table 4), it is lower than our standard (0.45), consequently, we hope we can give a plan to help Nepal to become more sustainable. The whole plan and the policies or programs involved have been shown below:



Figure 2: CO<sub>2</sub> emmissions, Life expectancy at birth and GNI per capita of Nepal [8]

- **GNI per capita :**  
From the figure 2, we can learn that GNI per capita growth in Nepal is relatively

slow in early years than in recent years. Considering that GNI per capita can be an index to show the general economical ability in a nation, we may deduce that the economic status is not so good in Nepal, as a result we give the follow policies or programs to enhance its economics:

1. We will introduce foreign investment to enhance our economics. Specifically, we will reduce the tax of foreign factory or company so as to appeal foreign investment.
2. We will set up program to encourage new projects or company in order to have more chances of work.
3. We will introduce some technical specialists by asking aid from ICM to develop our science and technology.

Above all, our goal in 2030 is to increase the GNI per capita with applying the policies and programs above.

- **The problem about  $CO_2$  emissions:**

The  $CO_2$  emissions in Nepal is relatively lower than other low income country, considering this fact, we recommend Nepal can take economic development as priority. But this do not means we can ignore the  $CO_2$  emissions, the government of Nepal should control their  $CO_2$  emission in a relative low status. Specifically, we hope the  $CO_2$  emissions index( defined as  $CO_2$  emissions divide GDP) in 2030 will not be higher than now.

- **Health situation:**

With the statistics shown in the figure 2, the life expectancy at birth in Nepal is longer than South Asia and other low income countries in recent years, but we cannot casually draw the conclusion that we can temporarily pay few attention to the health status in Nepal. In fact, there are still some problems need to be settled such as food security, public health system, accessing to the improved water etc. We should not develop economics at the cost of peoplea's health, consequently, we have the following policies or programs for Nepal to choose:

1. Recruiting some medical staff from other countries and introducing advanced managing system to arrange health system in Nepal.
2. Buying more water purifying machines and distributing them around country will increase the number of people who can get improved water.
3. Posting policies that nation will buy medical insurance for people over 60 years old. Considering the economics of Nepal, we can set this policies as a long term goal.
4. Inviting the staff of WHO (World Health Organization) to organize some lectures about protection from common disease.

By considering the policies and programs mentioned above, we estimate that the life expectancy at birth will increase over 20 years, meanwhile the percentage of population can access to the improved water in the total population will rise over 20 years.

- **Promoting about energy:** By analyzing the data from World Bank, we can see that the percentage of alternative energy and nuclear energy in the total energy use in Nepal is relatively lower than other countries. In order to popularize the alternative energy and nuclear energy, we can announce programs to build more hydropower station, wind power station and solar power station etc. In the meantime, we can ask help from the ICM such like technical staff and relative devices. With this program, we can achieve the long term goal: the percentage of alternative energy and nuclear energy in the total energy use will rise over 20 years.
- **Forest:** Through the data from World Bank, we know that the percentage of forest area in the land area is relatively stable in past years, but considering the fact that we cannot avoid damaging forest in the process of development, we have to give some policies to control the negative effects on forest, and give some programs to increase this percentage:
  1. The supervision department should periodically inspect the negative effect on environment (include forest) caused by the activities of companies and institution and giving a report to limit their behaviors which may violate environment.
  2. Promulgating program that the spare farmland will be used to plant trees with the permission of it owner.
  3. Promulgating program to change desertified land and salinized land into normal land. In this process, asking help from other developed countries is necessary.

By applying the policies or programs above, we hope the percentage of forest area in total land area will increase.

- **Ecological Footprint:** The ecological footprint of Nepal in the past few years (2007-2010)—(0.74 0.74 0.74 0.82)[9] is relatively lower than other countries. This means that demand for the natural resource of individual is relatively lower. This is a good news for the reason that according to the EF index the land and the resource of Nepal is abundant enough to feed more people. In order to keep this trend, we can keep the present policies. We hope the EF index in 2030 will not be higher than now.

## 4 Prediction and optimization

In this section, we will establish our prediction and optimization model based on regression model[7].

### 4.1 Prediction of our plan for Nepal

In order to get the prediction of Nepal over 20 years, we have to establish a prediction model to measure this change.

Noticing that the data of 7 indicators of Nepal in the past 15 years can be fitted by liner function or exponential function(the process can be realized by matlab). If we apply  $\log$  function on the independent variable of exponential function, the exponential function could become liner function.

Consequently, we only need to predict a sequence of number which has a liner relationship with time. To realize this goal, we can use the regression model.

We use following equation to predict the change of each indicator:

$$Y(\text{or } \log Y) = \beta_0 + \beta_1 T + \varepsilon \quad (6)$$

**The meaning of parameters in the formula (6):**

- $Y(\text{or } \log Y)$  means the value of the indicators.
- $\beta_0, \beta_1$  is the regression coefficients.
- $T$  represent time.
- $\varepsilon$  means random deviation include the influence brought by natural disasters and war etc. Especially,  $\varepsilon$  is a random variable which has normal distribution. (We will assume these in the assumption later.)

**In order to illustrate our model, we may need these assumption:**

- In the model of forecasting the change of 7 indicators over the 20 years in Nepal, we assume that all events of small probability such like natural disasters, war are concluded in one factor  $\varepsilon$  to show the influence caused by those unpredictable factors. Here we assume that  $\varepsilon$  has a normal distribution  $N(0, \sigma^2)$ .

**Note:**

We choose  $\varepsilon$  which has a normal distribution  $N(0, \sigma^2)$  because we use regression model to predict the change of 7 indicators in Nepal, and regression model usually use a random variable which has a normal distribution to show the random deviation.

- Considering the fact that once the factors which we put in  $\varepsilon$  happen, the influence over our model will be destructive. Since these factors all have a small probability, the average effect of the influence in a long term will not be so uncontrollable. So assuming the  $\sigma$  has a small value is reasonable.
- We assume that the plan aided by ICM only influence the value of  $\beta_1$
- We assume that Nepal will naturally develop in a constructive way. It means that its score calculated by our sustainable development model over 20 years will increase naturally even without the aiding plan.
- We assume the indicator's value  $V_i$  have the form of formula (6):

$$V_i(\text{or } \log V_i) = \beta_{i,0} + \beta_{i,1} T + \varepsilon \quad (i = 1, 2 \dots 7) \quad (7)$$

In other words, the value of  $V_i$  only have relationship with:

1.  $\varepsilon$  : refers to the environmental factors assumed before.
2.  $T$  : means the time.
3.  $\beta_0, \beta_1$  :  $\beta_0$  and  $\beta_1$  refers to the regression coefficients, and  $\beta_1$  will be influenced by our plan and the aid form ICM.

**Note:** This assumption is reasonable and we will explain it specifically in each indicators later.

According to the theoretical analysis at the beginning of this subsection, we can use formula (6) to fit the 7 indicators and predict their change:

- **GNI per capita** : It is reasonable to presume that the growth of GNI per capita can be described by exponential function because the increasing in GNI can stimulate technique boost, and this boost can accelerate growth rate of economics. This fitting can also be reflected by the data between 1996 and 2010.
- **Access to an improved water source(% of rural population with access)** : We assume the amount of facilities to provide clean water increase in a stable rate. So it can be describe by a linear function.
- **Ecological Footprint** : As the value of Ecological Footprint is extremely unstable and can be influenced by many factors, we only focus on its trend and describe it by a linear function.
- **CO<sub>2</sub> emissions index** : As the development of the environmental-friendly technology, we can create more economic effects with less CO<sub>2</sub> to emit. The CO<sub>2</sub> emissions index stands for the amount of CO<sub>2</sub> generated to produce a defined amount of product.
- **Life expectancy at birth:** Considering the fact that longevity of human has a limitation, so, we use  $100 - ae^{-bx}$  to fit those dots.
- **Forest area (% of land area):** Considering the fact that the hewing rate and the growing rate of forest will finally strike a balance, so, we use  $a + be^{-cx}$  to fit those dots.
- **Alternative and nuclear energy (% of total energy use):** Assume that once the old devices using dirty energy are completely damaged, we replace it with devices using clean energy. As the longevity of most machines obey exponential distribution. So the clean energy dots can be fitted by  $1 - ae^{-bx}$ .

By using the matlab to calculating the regression coefficient and simulating the regression, we give the graph of the change of different indicators. The fitting results are shown in Figure 3.

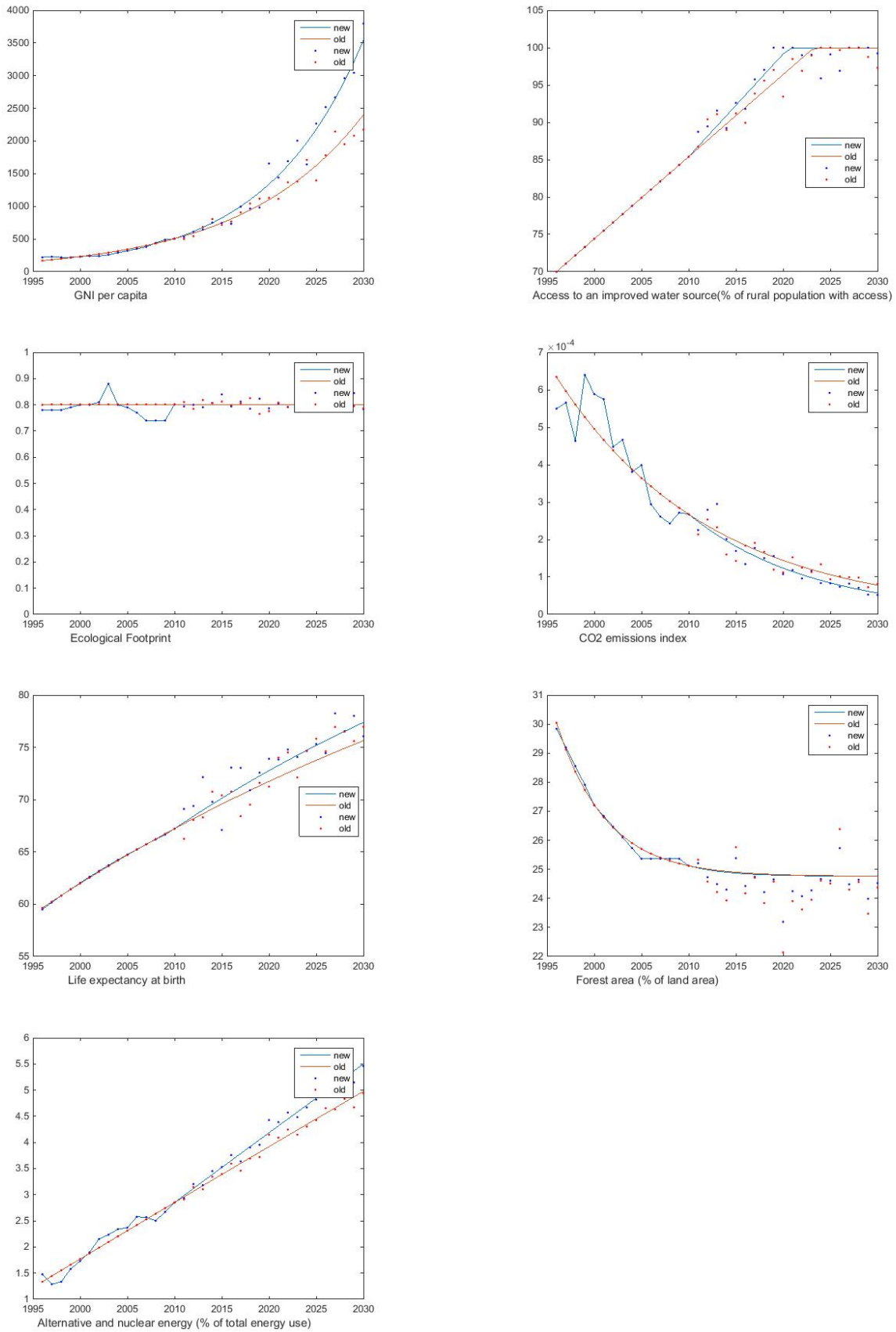


Figure 3: The change of the seven indicators

The blue line in the graph shows the predicting development over 20 years by applying our plan, the red line in the graph show the natural development over 20 years without our plan. The blue dots mean the influence brought by the random deviation  $\varepsilon$  with our plan. The red dots mean the influence brought by the random deviation  $\varepsilon$  without our plan.

From the Figure 3, we can clearly see the change that Nepal become more sustainable between 20 years with our plan or without our plan.

## 4.2 How to select a effective plan for ICM

### 4.2.1 Assumption

- We assume that ICM only ameliorate one indicator each year.
- We assume that the growth rate in each indicator will return to a normal status after ICM end the aid.

### 4.2.2 Select a highly effective plan for ICM

According to our assumptions, ICM can only help to improve one indicator. In other words, ICM will decide one of the most valuable indicator to investigate. It's hard to choose when there are multiple indicators. Fortunately, we develop a efficient algorithm based on our prediction model.

Before introducing the optimization algorithm, we define the concept of *the most valuable*. When we judge whether a plan is of great efficiency from the growth in the score by applying the plan, the plan which improve the score most is defined as the most valuable one.

The abstract process of our algorithm has been shown in Figure 4.



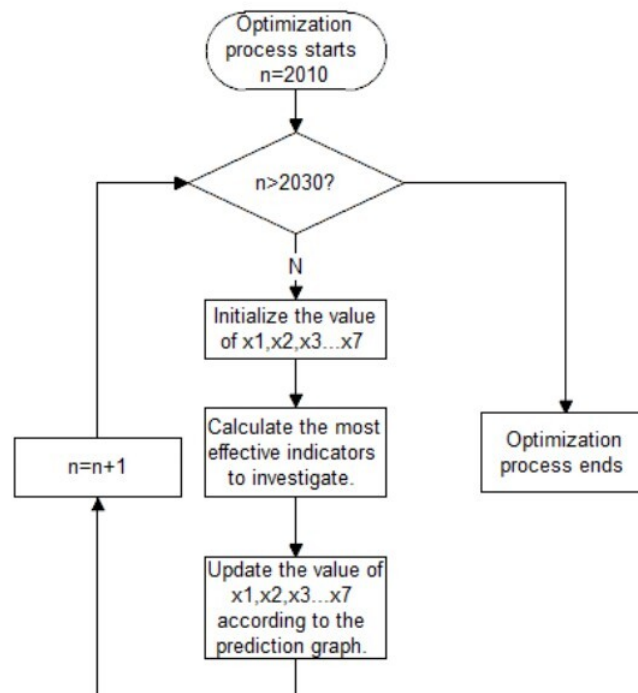


Figure 4: Optimization process

The  $n$  represent the present year,  $x_i$  shows the value of the  $i$  th indicator's value at present. The predicting graph is shown in figure 3. The calculating to select the most effective plan is by comparing the growth of the score (using our sustainable development model in §2.3.2). Because ICM aid to boost the growth rate (the regression coefficient  $\beta_1$ ) and the final score will change. By comparing the 7 different scores we find the highest one as the best choice each year.

We finally calculate the most effective plan and scores over 20 years as follow :

Table 5: The most effective plan and scores

year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
$i$	4	4	4	4	4	4	4	4	4	4
score	0.3506	0.3612	0.3716	0.3817	0.3915	0.4012	0.4106	0.4199	0.4289	0.4378
year	2021	2022	2023	2024	2025	2026	2027	2028	2029	2030
$i$	7	7	7	1	1	1	1	1	1	1
score	0.4465	0.4551	0.4636	0.4719	0.4801	0.4882	0.4963	0.5042	0.5121	0.5198

## 5 Strength and weakness

We build several models to solve the problem and inevitably, all of them have their own strength and weakness.

## 5.1 Sustainable development model

### Strength:

- The AHP method we use are really mature. By applying with both quantitative and qualitative analysis, it has been examined to be very useful when people face this kind of abstract problem in different subjects. So applying AHP method in our sustainable development model is very suitable.
- The result in our model simulation shows that our model and scoring system is good enough to identify whether a country is sustainable or not.
- By using the data of United States in 2007 as standard, we avoid the physical dimension problem in scoring. And that makes our model more reliable.
- Our indicators include many aspects such as economics, environment, citizen's health and sanitary situation. We think the indicators can describe the problem from different aspects.

### Weakness:

- We assume that the seven indicators are independent but in fact, some indicators may influence others. The  $CO_2$  index can influence the EF index, and the percentage in the AE index can somehow influence life expectancy.
- We only get the graph of EF index instead of specific data from the Global Footprint Network website and we build a grid to get the data of every year from the graph. The data we get may not accurate enough, but still persuasive. So we consider this situation as part of our weakness.
- Although our indicators are from different aspects, the number of our indicators are not enough to elaborate the problem. Therefore, we find a polarization phenomenon which refers to the phenomenon that some countries like France get considerably higher scores, while some countries get a quite lower score.

## 5.2 Prediction and Optimization model

### Strength:

- The results of our model basically conform to the fact.
- We use  $\varepsilon$  to represent the random deviation caused by natural disasters and war etc.
- Regression model need numerous data to analysis and we use the data of 15 years, so the result can be rational.

### Weakness:

- We don't consider the environment factors concretely and we conclude those events of small probability in one factor. This does make sense, but it is not accurate enough.
- We try not to put up so many assumptions, but considering the data limit, we still have lots of assumptions in our models. That makes our model not useful in some special cases.

## 6 Conclusions

Sustainable development has become an important issue recent years. Without quantitative and qualitative analysis, we can't evaluate the sustainability of a country. By applying our sustainable development model which is based on AHP model, we can easily calculate the "sustainable score" of a country. Our model testing results show that we can clearly distinguish more sustainable countries from less sustainable ones. So our model can serve to inform the ICM on those countries that need the most support and intervention.

Additionally, we calculate Nepal's score by using our sustainable development model and find out that Nepal is not very sustainable. So we give a plan consisting of several programs, policies and aids from ICM in order to help Nepal become more sustainable.

Then we predict the change that will occur by implementing our plan with further assumption. In this process, we use regression analysis with a small random perturbation and make the graph of each indicator's change.

Finally we assume that ICM only investigate in one indicator each year, and we use our prediction and optimization model to give an optimized strategies for ICM.

## References

- [1] Bell, Simon and Stephen Morse. 2008. Sustainability Indicators: measuring the immeasurable. Earthscan, London.
- [2] "Gross national income" Wikipedia  
[http://en.wikipedia.org/wiki/Gross\\_national\\_income](http://en.wikipedia.org/wiki/Gross_national_income)
- [3] "Footprint Basics - Overview"  
[http://www.footprintnetwork.org/en/index.php/GFN/page/footprint\\_basics\\_overview/](http://www.footprintnetwork.org/en/index.php/GFN/page/footprint_basics_overview/)
- [4] Saaty, Thomas L. 1982. Strategy and Organization, The Analytical Hierarchy Process for Decisions in a Complex World. Belmont, CA: Lifetime Learning Pub.
- [5] Qiyuan Jiang, Jinxing Jie, Yejun 2010. Mathematical Model. Higher Education Press of China. Page 254
- [6] World Bank  
<http://data.worldbank.org>
- [7] L. A. ZADEH. 1965. Information and Control, Fuzzy Sets. Page 338–353
- [8] "Nepal" World Bank  
<http://data.worldbank.org/country/nepal?display=default>
- [9] "Ecological Footprint of Nepal" Footprint Network  
<http://www.footprintnetwork.org/en/index.php/GFN/page/trends/nepal/>
- [10] Robert Costanza a,\* , Bernard C. Patten b. Ecological Economics 15(1995). Defining and predicting sustainability. Page 193–196.
- [11] Daniel D. Morana,b, Mathis Wackernagel, Justin A. Kitzes, Steven H. Goldfinger, Aurélien Boutaud. Ecological Economics 64(2008). Measuring sustainable development–Nation by nation. Page 470–474